

Deuteron Spin Structure Functions in the Resonance and DIS Regions

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Abstract

We derive relations between spin-dependent nuclear and nucleon g_1 and g_2 structure functions within the nuclear impulse approximation, which are valid at all Q^2 , and in both the resonance and deep inelastic regions. We apply the formalism to the specific case of the deuteron, which is often used as a source of neutron structure information, and compare the size of the nuclear corrections calculated using exact kinematics and using approximations applicable at large Q^2 .

1. Introduction

The study of nuclear effects in deep inelastic structure functions has by now a long and rich history. The importance of nuclear structure in high-energy scattering of leptons from nuclei was most prominently thrust into the limelight by the “nuclear EMC effect” discovered by the European Muon Collaboration (EMC) [1], which found much larger than anticipated differences between structure functions of heavy nuclei and those of deuterium. Many theoretical and experimental studies of nuclear effects on structure functions followed, and over the years an extensive phenomenology has been developed, even though many questions about the origin of the effect still remain (for reviews see *e.g.* Ref. [2]).

While the early studies of nuclear medium modifications focussed on heavy nuclei, where the magnitude of the effects is largest, it has since been realized that resolving the dynamical origin of the EMC effect requires understanding of light nuclei as well. Until recently, an anomalous situation existed whereby nuclei with $A \lesssim 4$, for which the most detailed microscopic theoretical calculations were possible, had the least empirical information available. A dedicated experiment at Jefferson Lab [3], with the goal of making precise measurements of the nuclear dependence of structure functions in a variety of light nuclei, has recently completed taking data which are currently being analyzed.

Light nuclei, such as deuterium or ^3He , are also often used as effective neutron targets in experiments seeking to extract information on the structure of free neutrons. This has been especially relevant for neutron spin structure studies, for which polarized ^3He is commonly used. If one is to extract reliable information on the structure of the neutron, it is important that the nuclear corrections are properly accounted for, especially given the ever increasing precision of modern experiments.

In recent years several theoretical analyses have been devoted to the nuclear EMC effect in $A = 2$ and $A = 3$ nuclei, both for unpolarized [4-16] and polarized [17-25] structure functions, which have quantified the effects of binding, Fermi motion, as well as relativity and nucleon off-shell effects. Most of these studies have focussed on nuclear effects in the deep inelastic scattering (DIS) region, where the exchanged photon’s virtuality Q^2 and the mass W of the final hadronic state are both large ($Q^2, W^2 \gg 1 \text{ GeV}^2$).

On the other hand, considerable data have been accumulated recently for $W \lesssim 2 \text{ GeV}$, in the region dominated by nucleon resonances. The resonance region has received renewed

interest partly due to the remarkable phenomenon of Bloom-Gilman duality [26], in which averages of resonance structure functions have been found to approximately equal the scaling region functions, measured at much larger W (see Ref. [27] and references therein). Extracting information on the neutron in the resonance region from nuclear data is particularly challenging.

The resonance region has considerably richer structure because of the prominence of specific resonances in the inclusive spectrum, such as the $\Delta(1232)$ or the $S_{11} N^*(1535)$ resonances. As well as exhibiting pronounced peaks, these resonance structures are also strongly Q^2 dependent. At high Q^2 the role of resonances is restricted by kinematics to the $x \sim 1$ domain, however, at low Q^2 ($\sim 1 \text{ GeV}^2$) resonances actually dominate the cross section. The effects of nucleon Fermi motion are expected to wash out much of these structures in a nucleus, leading to dramatic differences between structure functions of nucleons and nuclei, and hence much more interesting EMC effects than in the deep inelastic region.

The theoretical tools for the study of nuclear effects in structure functions have, for historical reasons, been developed specifically for the DIS region, usually assuming the Bjorken limit (where both Q^2 and the energy transfer to the target $\rightarrow \infty$), in which the kinematics greatly simplify. To date, however, only approximate methods have been used to describe nuclear corrections in the resonance and low- Q^2 regions, mostly using either effective polarizations, or convolution approximations which are valid strictly only in the Bjorken limit.

In this paper we rectify this problem. Working within the nuclear impulse approximation, in which the virtual photon interacts with a single nucleon inside the nucleus, we derive a set of formulas for the spin-dependent nuclear g_1 and g_2 structure functions, which are valid over a broad range of kinematics. In fact, since the derivation involves an exact treatment of finite Q^2 kinematics, and does not depend on the twist expansion, our results are valid for any Q^2 , and in both the deep inelastic and resonance regions (and even for real photoproduction).

The general derivation is made within a relativistic framework; however, for practical applications we specialize to the weak binding limit (WBL), in which the nuclear matrix elements are systematically expanded in powers of nucleon momentum \mathbf{p}/M , where M is the nucleon mass. In this limit, we find that the nuclear g_1^A and g_2^A structure functions can be written as generalized convolution of the nucleon $g_{1,2}^\tau$ structure functions ($\tau = p, n$) and spin-dependent nucleon energy-momentum distributions in nuclei. At finite Q^2 , the $g_{1,2}^A$ functions receive contributions from both g_1^τ and g_2^τ , in contrast to the Bjorken limit results

which are diagonal for g_1 [19, 20, 24].

Our formal results are quite general and applicable to all nuclei. However, in view of the current importance of understanding of nuclear corrections in light nuclei, we demonstrate our formalism by applying it to the simplest nucleus, namely the deuteron. In Sec. 2 we outline the derivation of the nuclear structure functions in the weak binding limit, and present for the first time the complete set of formulas for polarized nuclear structure functions in terms of those of bound nucleons, valid at arbitrary Q^2 . At finite Q^2 we find a breakdown of the simple convolution expressions for the nuclear structure functions, in which all of the Q^2 dependence is absorbed into the nucleon structure functions, with the nucleon momentum distributions being functions of the nucleon momentum and energy only. In the generalized convolution that we derive at finite Q^2 , the nucleon momentum distributions depend in addition on the parameter $\gamma \equiv |\mathbf{q}|/q_0$, where \mathbf{q} and q_0 are momentum and energy transfer, respectively.

In Sec. 3 we apply the formalism to the specific case of the deuteron, and study the dependence of the finite- Q^2 nucleon momentum distributions on the parameter γ . The numerical results for the deuteron structure functions are presented in Sec. 4, where we focus in particular on the EMC effect in the nucleon resonance region, and contrast this with the effect for leading twist structure functions. Finally in Sec. 5 we summarize our results and discuss future work.

2. Nuclear structure functions in the weak binding limit

We begin our discussion of the nuclear structure function by summarizing the results within the framework of the relativistic nuclear impulse approximation, in which the scattering from the nucleus proceeds via the scattering from its individual nucleon constituents. Corrections to the impulse approximation, in the form of multiple scattering from nucleons, or meson-exchange currents, are typically confined to the small- x region, and can be safely neglected by restricting the analysis to $x \gtrsim 0.2$. Possible corrections due to the final state

interaction of the spectator nucleon with the hadronic debris are also not considered here.

2 a. Hadronic tensor

In the impulse approximation the hadronic tensor for a nucleus with four-momentum P_A and spin S can be expressed in terms of the nucleon propagator in a nucleus and the virtual photon Compton amplitude for a bound nucleon (for more details see *e.g.* Refs. [8, 9, 16, 21, 22]):

$$W_{\mu\nu}^A(P_A, q, S) = \int [dp] \text{Tr} [\mathcal{A}^\tau(p, P_A, S) \mathcal{W}_{\mu\nu}^\tau(p, q)], \quad (1)$$

where the sum is taken over bound protons and neutrons ($\tau = p, n$), and the integration is performed over the nucleon four-momentum p , for which we use the contracted notation $[dp] = d^4p/(2\pi)^4$. In Eq. (1), $\mathcal{A}^\tau(p, P_A, S)$ is the imaginary part of the proton ($\tau = p$) or neutron ($\tau = n$) propagator in a nucleus A with momentum P_A and spin S :

$$\mathcal{A}_{\alpha\beta}^\tau(p, P_A, S) = \int dt d^3\mathbf{r} e^{i(p_0 t - \mathbf{p} \cdot \mathbf{r})} \langle P_A, S | \bar{\Psi}_\beta^\tau(t, \mathbf{r}) \Psi_\alpha^\tau(0) | P_A, S \rangle, \quad (2)$$

with $\Psi_\alpha^\tau(t, \mathbf{r})$ the (relativistic) nucleon field operator, and α and β Dirac spinor indices. The bound (off-shell) nucleon electromagnetic tensor $\mathcal{W}_{\mu\nu}^\tau(p, q)$ in Eq. (1) is a matrix in Dirac space, and the trace “Tr” is taken in the nucleon Dirac space.

The analysis of Eq. (1) in the fully relativistic case requires the solution to the nuclear bound state problem [in particular the calculation of $\mathcal{A}^\tau(p, P_A, S)$], a task yet to be fully completed.¹ The presence of nucleon spin introduces some complications in Eq. (1), such as additional Lorentz–Dirac structures (structure functions) in the hadronic tensor $\mathcal{W}_{\mu\nu}^\tau(p, q)$ for the bound nucleon [8, 9]. Nevertheless, the analysis can be significantly simplified in the target rest frame within a nonrelativistic approximation, or *weak binding limit* (WBL), assuming typical nucleon momenta and energies to be small compared to the nucleon mass.

Starting from the general expression for $\mathcal{W}_{\mu\nu}^\tau(p, q)$ for the off-shell nucleon, and performing a systematic expansion of the matrix elements in Eq. (1) in terms of $|\mathbf{p}|/M$ and ε/M up to order \mathbf{p}^2/M^2 and ε/M , where $\varepsilon = p_0 - M$ is the nucleon separation energy, one can show [9, 16, 22] that the nuclear hadronic tensor can be written in terms of the (nonrelativistic)

¹ We refer in this context to calculations based on Bethe-Salpeter and light-cone approaches [12, 28].

nucleon spectral function in the nucleus, $\mathcal{P}^\tau(\varepsilon, \mathbf{p}, S)$, and the bound nucleon hadronic tensor,

$$\frac{1}{M_A} W_{\mu\nu}^A(P_A, q, S) = \int [dp] \frac{1}{M + \varepsilon} \text{tr} [\mathcal{P}^\tau(\varepsilon, \mathbf{p}, S) \mathcal{W}_{\mu\nu}^\tau(p, q)] . \quad (3)$$

Here M_A is the nuclear mass, and the trace “tr” is taken over nucleon polarization states.

The spin-dependent part of the nuclear hadronic tensor is related to the antisymmetric part of the off-shell nucleon tensor $\mathcal{W}_{\mu\nu}^\tau$, which depends on the nucleon polarization. In the WBL this component of the hadronic tensor of the nonrelativistic bound nucleon is given by:

$$\mathcal{W}_{\mu\nu}^\tau(p, q) = \frac{M}{p \cdot q} \left[(g_1^\tau + g_2^\tau) \varepsilon_{\mu\nu\alpha\beta} q^\alpha \hat{\mathcal{S}}^\beta - g_2^\tau \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \frac{\hat{\mathcal{S}} \cdot q}{p \cdot q} \right] , \quad (4)$$

where $g_{1,2}^\tau$ are the spin structure functions of the off-shell proton or neutron ($\tau = p, n$) with four-momentum $p = (M + \varepsilon, \mathbf{p})$. The operator $\hat{\mathcal{S}}$ in Eq. (4) has a structure similar to that of the spin four-vector $(0, \boldsymbol{\sigma})$ boosted to a reference frame in which the nucleon has (nonrelativistic) momentum \mathbf{p} :

$$\hat{\mathcal{S}} = \left(\frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{M}, \boldsymbol{\sigma} + \frac{\mathbf{p}(\boldsymbol{\sigma} \cdot \mathbf{p})}{2M^2} \right) . \quad (5)$$

In Eq. (3) the nuclear hadronic tensor $W_{\mu\nu}^A$ factorizes into high-energy ($\mathcal{W}_{\mu\nu}$) and low-energy (\mathcal{P}) domains in the nuclear scattering amplitudes. The presence of spin, however, implies that this factorization does not in general carry over to a corresponding factorization of nuclear structure functions [8, 9, 21, 22], unless further assumptions or simplifications are made.

The relation between the nuclear and the nucleon spin structure functions for polarized ^3He and for the deuteron was discussed in Refs. [19, 20, 23, 24]. Note that the implementation of the impulse approximation is different in Ref. [19] and in the present approach (see also Ref. [22]). Here we begin from a relativistic impulse approximation, Eq. (1), which fully takes into account the nucleon spin degrees of freedom, and work within the WBL to obtain the nuclear hadronic tensor in terms of the nuclear spectral function, Eq. (3). The latter equation is similar to the corresponding relation in Ref. [19], but is not identical due to a different treatment of the reaction kinematics. In particular, in Ref. [19] the struck nucleon is assumed to be on the mass shell, while here we take the nucleon to be off-mass-shell.

Further details of the derivation of Eq. (3) will be given elsewhere [29].

2 b. Spectral function

The nuclear spectral function \mathcal{P} is defined similarly to the nucleon propagator in the nucleus \mathcal{A} in Eq. (2), but involves the correlator of nonrelativistic two-component nucleon operators:

$$\mathcal{P}_{\sigma\sigma'}^\tau(\varepsilon, \mathbf{p}, \mathbf{S}) = \int dt d^3r d^3r' e^{i(\varepsilon t - \mathbf{p} \cdot (\mathbf{r}' - \mathbf{r}))} \langle \psi_{\sigma'}^{\tau\dagger}(\mathbf{r}', t) \psi_\sigma^\tau(\mathbf{r}, 0) \rangle, \quad (6)$$

where the nucleon operator ψ_σ^τ describes the nucleon with polarization σ and isospin τ , and the average is taken over the nuclear ground state. The nuclear spin is described in Eq. (6) by the axial three-vector \mathbf{S} . For spin-1/2 targets, \mathbf{S} is simply the target polarization vector. For the spin-1 case \mathbf{S} is defined in terms of polarization vectors \mathbf{e}^m as $\mathbf{S} = i\mathbf{e}^{m*} \times \mathbf{e}^m$, where $m = 0, \pm 1$ is the spin projection along the axis of quantization.

The nuclear spectral function can in general be calculated by inserting a complete set of intermediate states and computing the resulting transition matrix elements between the ground and intermediate states. For the deuterium nucleus, the intermediate states are exhausted by a single proton or neutron, and the spectral function is expressed entirely in terms of the deuteron wave function. However, already for $A = 3$ nuclei the calculation of the spectral function is considerably more complicated [19, 20].

The general spin structure of the spectral function can be obtained by expanding the matrix $\mathcal{P}_{\sigma\sigma'}^\tau$ in terms of the Pauli spin matrices and applying constraints from parity and time reversal invariance. One can then write the spectral function in the general form (for both the proton and neutron contributions) [19]:

$$\mathcal{P}^\tau(\varepsilon, \mathbf{p}, \mathbf{S}) = \frac{1}{2} (f_0^\tau \mathbf{I} + f_1^\tau \boldsymbol{\sigma} \cdot \mathbf{S} + f_2^\tau T_{ij} S_i \sigma_j), \quad (7)$$

where \mathbf{I} is the unity matrix and $T_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$ is a traceless symmetric tensor with $\hat{p}_i = p_i/|\mathbf{p}|$ for any component i of the momentum (the sum over repeated indices is implied).

Since the spectral function is hermitian, the coefficients f_i^τ in Eq. (7) are real functions of the energy ε and momentum \mathbf{p} . The function f_0^τ gives the spin averaged spectral function, and is normalized to the number of protons ($\tau = p$) or neutrons ($\tau = n$) in the nucleus:

$$\int [dp] \text{tr} [\mathcal{P}^{p(n)}(\varepsilon, \mathbf{p}, S)] = \int [dp] f_0^{p(n)} = Z (A - Z). \quad (8)$$

The functions f_1^τ and f_2^τ describe the distribution of nuclear spin amongst the nucleons. The integrated functions f_1^τ and f_2^τ determine the average nucleon polarization in the target and the average tensor polarization, respectively:

$$\langle \sigma_z \rangle^\tau = \int [dp] f_1^\tau, \quad (9)$$

$$\langle T_{zi} \sigma_i \rangle^\tau = \frac{2}{9} \int [dp] f_2^\tau, \quad (10)$$

where we take the nuclear spin vector to lie along the z -axis.

2 c. Master formula

Projecting from the hadronic tensor (3) the appropriate structure functions, we obtain our “master formula” for the g_1^A and g_2^A structure functions of the nucleus:

$$x g_a^A(x, Q^2) = \int [dp] D_{ab}^\tau(\varepsilon, \mathbf{p}, \gamma) x' g_b^\tau(x', Q^2, p^2), \quad (11)$$

where $a, b = 1, 2$, and again summation over repeated indices is implied. In Eq. (11) $x' = Q^2/2p \cdot q = x/(1 + (\varepsilon + \gamma p_z)/M)$ is the Bjorken variable for the bound nucleon, with $x = Q^2/2Mq_0$, and $\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2 x^2/Q^2}$ takes into account finite- Q^2 kinematics. The spin-dependent nucleon momentum distribution functions D_{ab} can be written in terms of the coefficient functions f_i (dropping isospin labels) as:

$$D_{11} = f_1 + \frac{3 - \gamma^2}{6\gamma^2} (3\hat{p}_z^2 - 1) f_2 + \frac{v\hat{p}_z}{\gamma} \left(f_1 + \frac{2}{3} f_2 \right) + v^2 \frac{(3 - \gamma^2)\hat{p}_z^2 - 1 - \gamma^2}{12\gamma^2} (3f_1 - f_2), \quad (12a)$$

$$D_{12} = (\gamma^2 - 1) \left[-\frac{3\hat{p}_z^2 - 1}{2\gamma^2} f_2 + \frac{v\hat{p}_z}{\gamma} \left(f_1 + \left(\frac{3}{2}\hat{p}_z^2 - \frac{5}{6} \right) f_2 \right) - v^2 \left(\frac{1 + \hat{p}_z^2(4\gamma^2 - 3)}{4\gamma^2} f_1 + \frac{5 + 18\hat{p}_z^4\gamma^2 - 5\hat{p}_z^2(3 + 2\gamma^2)}{12\gamma^2} f_2 \right) \right], \quad (12b)$$

$$D_{21} = -\frac{3\hat{p}_z^2 - 1}{2\gamma^2} f_2 - \frac{v\hat{p}_z}{\gamma} \left(f_1 + \frac{2}{3} f_2 \right) - v^2 \frac{3\hat{p}_z^2 - 1}{12\gamma^2} (3f_1 - f_2), \quad (12c)$$

$$D_{22} = f_1 + \frac{2\gamma^2 - 3}{6\gamma^2} (3\hat{p}_z^2 - 1) f_2 + \frac{v\hat{p}_z}{\gamma} \left[(1 - \gamma^2)f_1 + \left(-\frac{5}{6} + \frac{\gamma^2}{3} + \hat{p}_z^2 \left(\frac{3}{2} - \gamma^2 \right) \right) f_2 \right] + v^2 \left[\frac{\hat{p}_z^2(3 - 6\gamma^2 + 4\gamma^4) - 1 - 2\gamma^2}{4\gamma^2} f_1 \right]$$

$$+ \frac{5 - 2\gamma^2(1 + 3\hat{p}_z^2) + 4\hat{p}_z^2\gamma^4}{12\gamma^2}(3\hat{p}_z^2 - 1)f_2 \Big] , \quad (12d)$$

where the velocity parameter $v = |\mathbf{p}|/M$.

The derivation of Eqs. (12) makes use of the fact that characteristic values of v are small, and one can expand the kinematical factors in a series in v keeping terms up to $\mathcal{O}(v^2)$. The functions D_{ab} have also been averaged over the polar angle of the nucleon momentum in the transverse plane (p_x, p_y) using the independence of $f_{1,2}$ of the directions of the nucleon momentum and the independence of the Bjorken variable x' of p_x and p_y .

In general the structure functions $g_{1,2}^\tau(x', Q^2, p^2)$ of the bound nucleon are functions of the invariant mass squared of the nucleon, since $p^2 \neq M^2$. This dependence is *a priori* unknown, but has been estimated in various models [8, 9, 15, 16, 22]. In the WBL, however, and especially for light nuclei such as deuterium, the degree to which the nucleons are off-mass-shell is not large, and we can assume that the off-shell functions can be approximated by their on-shell values, $g_{1,2}^\tau(x', Q^2, p^2) \approx g_{1,2}^\tau(x', Q^2)$.

Note that while the above derivation is valid in the WBL ($|\mathbf{p}|, |\varepsilon| \ll M$), Eq. (11) holds for arbitrary momentum transfer q . An interesting feature of the distribution functions D_{ab} is that they depend on the momentum transfer through the dimensionless parameter γ . Furthermore, nuclear effects cause the off-diagonal distributions D_{12} and D_{21} to be nonzero, which results in mixing of the g_1 and g_2 structure functions in the convolution integral (11). In leading order in v the functions D_{12} and D_{21} are driven by tensor distribution $(3\hat{p}_z^2 - 1)f_2$. In the limit of high Q^2 , the parameter $\gamma \rightarrow 1$ and the distributions (12) simplify considerably. In particular, $D_{12} \rightarrow 0$ and the convolution formula for g_1 becomes diagonal (*i.e.* there are no contributions from g_2^τ to g_1^A). However, mixing in g_2^A persists even in this limit [22].

A similar mixing of the nucleon structure functions g_1^τ and g_2^τ in the g_1^A nuclear structure function was observed in Refs. [19, 20, 24]. However, the distribution functions in Eq. (12) are different from the corresponding results in Refs. [19, 20] in the higher order terms in v and $\gamma^2 - 1$. As discussed at the end of Sec. 2a, these differences arise from the different treatments of the impulse approximation here and in Refs. [19, 20, 24].

The fact that Eq. (11) can be applied to both the inelastic and quasi-elastic scattering, at any Q^2 , allows us calculate nuclear structure functions in the DIS region (at large Q^2 and W), as well as at lower Q^2 , where the low- W resonance region plays a more prominent role. In this context we note that the dependence of the effective nuclear distributions

Eq. (12) on Q^2 enters through the dimensionless parameter γ , a feature which allows a simple parameterization of the effective distributions over the full range of kinematics [29]. In the next section we apply the formal results presented here to the specific case of the deuteron.

3. Deuteron spin structure functions

For the case of lepton scattering from a deuteron, the functions f_i in Eq. (7) can be written in terms of the deuteron wave functions as (see also Appendix B of Ref. [22]):

$$f_0 = 4\pi^3 (\psi_0^2 + \psi_2^2) \delta(\varepsilon - \epsilon_D + \mathbf{p}^2/2M) , \quad (13a)$$

$$f_1 = 4\pi^3 (\psi_0^2 - \psi_2^2/2) \delta(\varepsilon - \epsilon_D + \mathbf{p}^2/2M) , \quad (13b)$$

$$f_2 = 4\pi^3 \frac{3}{2}(\psi_2^2 - \sqrt{2}\psi_0\psi_2) \delta(\varepsilon - \epsilon_D + \mathbf{p}^2/2M) , \quad (13c)$$

where $\epsilon_D = -2.2$ MeV is the deuteron binding energy, and ψ_0 and ψ_2 are the S - and D -state momentum space wave functions, respectively, normalized such that:

$$\int_0^\infty d\mathbf{p} \mathbf{p}^2 (\psi_0^2(p) + \psi_2^2(p)) = 1 . \quad (14)$$

Since the deuteron is an isoscalar nucleus, Eqs. (13) hold for both the proton and neutron distributions. The average nucleon polarization and tensor polarization in the polarized deuteron can then be expressed as:

$$\langle \sigma_z \rangle = 1 - \frac{3}{2}P_D , \quad (15)$$

$$\langle T_{zi} \sigma_i \rangle = \frac{1}{3}(P_D - \sqrt{2}P_{SD}) , \quad (16)$$

where $P_D = \int d\mathbf{p} \mathbf{p}^2 \psi_2^2(p)$ is the D -state probability, and $P_{SD} = \int d\mathbf{p} \mathbf{p}^2 \psi_0(p)\psi_2(p)$ is the S - D interference in the deuteron. For the deuteron wave function calculated from the Paris potential [30] one has $P_D = 5.8\%$ and $P_{SD} = 9.4\%$, while for the Bonn potential [31] $P_D = 4.3\%$ and $P_{SD} = 10.1\%$. Note that in the $\gamma \rightarrow 1$ limit, the distributions D_{ab} in Eqs. (12) for the deuteron are equivalent to those in Ref. [22], where the functions f_i were effectively defined including a factor $(1 - \mathbf{p}^2/2M^2)$, which in our notation is now contained inside the distributions D_{ab} .

As discussed in Sec. 2 above, at finite Q^2 the deuteron structure functions $g_{1,2}^d$ receive contributions from both the isoscalar g_1^N and g_2^N structure functions of the nucleon ($N =$

$p + n$) individually. In particular, while the contribution from g_2^N to g_1^d vanishes in the Bjorken limit, it is non-zero at finite Q^2 . To illustrate the relative importance of the $g_{1,2}^N$ contributions to the deuteron structure functions, we can compare the individual momentum distributions D_{ab} for the diagonal and off-diagonal terms, and also as a function of γ .

In the simplest convolution model the nuclear structure functions in Eq. (11) are written as convolutions of the nucleon structure functions and effective nucleon *light-cone* momentum distributions:

$$g_a^d(x, Q^2) = \int_x \frac{dy}{y} \tilde{D}_{ab}(y, \gamma) g_b^N\left(\frac{x}{y}, Q^2\right), \quad (17)$$

where $y = (p_0 + \gamma p_z)/M = (1 + (\varepsilon + \gamma p_z)/M) = x/x'$. In the Bjorken limit ($\gamma \rightarrow 1$) the variable y is the light-cone fraction of the deuteron carried by the interacting nucleon. The effective light-cone momentum distributions \tilde{D}_{ab} are obtained by integrating the functions D_{ab} in Eq. (12):

$$\tilde{D}_{ab}(y, \gamma) = \int [dp] D_{ab}(\varepsilon, \mathbf{p}, \gamma) \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right). \quad (18)$$

In Fig. 1 we show the nucleon light-cone momentum distributions $\tilde{D}_{ab}(y, \gamma)$ for several values of γ . The results for $\gamma = 1$ correspond to the Bjorken limit distributions. Note that in this limit the function \tilde{D}_{12} vanishes. The diagonal functions \tilde{D}_{11} and \tilde{D}_{22} are significantly larger than the off-diagonal functions, but decrease in magnitude for larger γ . On the other hand, the distribution \tilde{D}_{12} becomes larger with increasing γ , with its magnitude reaching $\sim 5\%$ that of \tilde{D}_{11} at $\gamma = 2$.

In Ref. [23] the resonance region was studied using a finite- Q^2 convolution formula for g_1^d similar to that in Eq. (17). However, the non-diagonal term arising from g_2^N was not present. We find that this term, although small, does make a non-zero contribution at finite values of Q^2 .

In the next section we use Eq. (11) to evaluate the effects of the smearing of the nucleon structure functions by the nucleon momentum distributions, and compare the calculated deuteron structure functions with the input nucleon structure functions in the resonance and deep inelastic regions.

4. Nuclear effects

In this section we present results for the g_1^d and g_2^d structure functions of the deuteron,

and compare these with the free nucleon functions. We focus in particular on the resonance region, with $W \lesssim 2$ GeV, which has to date received little attention. The need to understand nuclear effects in the deuteron g_1^d and g_2^d structure functions has arisen partly in response to the recent high-precision data on the spin dependent deuteron structure functions from Jefferson Lab [32, 33, 34].

For the nucleon g_1^τ and g_2^τ structure functions, in this analysis we consider the parameterizations from the MAID unitary isobar model for $(e, e'p)$ reactions [35], from Simula *et al.* [36], and from EG1 in CLAS at Jefferson Lab [37]. These parameterizations encompass the resonance as well as the deep inelastic regions. In the case of the MAID model, however, only a few selected final states are considered, so this parameterization is expected to underestimate the high- W (or low- x) region.

In Figs. 2 and 3 we show the input xg_1^N and xg_2^N structure functions, respectively, for an isoscalar nucleon ($N = p + n$), at a sample Q^2 value, $Q^2 = 2$ GeV². At this Q^2 the most prominent structure at large x in both the g_1^N and g_2^N structure functions is the peak associated with the P_{33} Δ resonance, which is negative for g_1^N and positive for g_2^N . The three models give qualitatively similar results here, although quantitatively there are some differences. At lower x the second and third resonance regions are also prominent, and here the MAID fit is smaller in magnitude, as expected, given that it is constructed primarily to describe low- W data.

For comparison in Fig. 2 we also show the leading twist (“BB”) parameterization of xg_1^N from Ref. [38], which is smooth and does not contain any resonance structure. It is interesting to observe that the leading twist fit appears to go through the average of the resonances (with the exception of the Δ resonance) for the MAID fit [35], reminiscent of the Bloom-Gilman duality between resonance and DIS structure functions [27]. The other resonance parameterizations [36, 37] are on average larger in magnitude than the leading twist curve, which may reflect the presence of the nonresonant background that is included in these fits.

Using the BB parameterization [38] we also calculate the leading twist Wandzura-Wilczek (WW) approximation to g_2 , $g_2^{\text{WW}}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 dy g_1(y, Q^2)/y$, which is shown in Fig. 3. This again displays very different behavior, both in magnitude and sign, compared with the g_2^N resonance parameterizations at large x . As with the g_1^N comparison, the leading twist curve appears to average the second and third resonance regions at intermediate x as

described by the MAID [35] fit, but is smaller in magnitude here compared with the other fits [36, 37].

With these nucleon structure function parameterizations, we can now investigate nuclear effects in both the resonance and DIS regions. For the resonance region ($W \lesssim 2$ GeV), we use the MAID parameterization [35] for g_1^N and g_2^N . The deuteron xg_1^d structure function, calculated using Eqs. (11), (12a) and (12b) with the Paris deuteron wave function, is shown in Fig. 4 at $Q^2 = 2$ GeV² (solid curve). Here the full calculation of xg_1^d is compared with that using the Bjorken limit in the momentum distributions D_{11} and D_{12} (dashed), and with the free nucleon structure functions (dotted). Where the resonance structures are clearly evident in the nucleon g_1^N functions, they are significantly diluted in the deuteron structure function. At the peak of the Δ resonance, for example, the magnitude of the deuteron g_1^d is about half that of the nucleon g_1^N . A similar effect is seen for the g_2^d structure function shown in Fig. 5.

The differences between the full results for $g_{1,2}^d$ and the Bjorken limit ($Q^2 \rightarrow \infty$) approximation for D_{ab} are small where the nucleon $g_{1,2}^N$ are smooth, but become more significant in the vicinity of the resonance peaks. At intermediate x , around the second and third resonance regions, the full results are some 10–15% smaller in magnitude than the Bjorken limit structure functions. At larger x the enhanced smearing is even more dramatic, with the full results being up to 25–30% smaller at the Δ resonance peak than those with the Bjorken limit smearing. This behavior can be understood from the nucleon light-cone distribution functions in Fig. 1, which decrease in magnitude with increasing γ . At fixed $Q^2 = 2$ GeV², the third resonance region at $x \approx 0.5$ corresponds to $\gamma \approx 1.5$, while for the Δ peak at $x \approx 0.75$ one has $\gamma \approx 2$. Therefore, at fixed Q^2 , larger x implies larger γ , and hence stronger smearing effects in both g_1^d and g_2^d .

The enhanced smearing at finite- Q^2 kinematics is also evident for leading twist structure functions, although the effects here are less dramatic. In Figs. 6 and 7 we show the deuteron xg_1^d and xg_2^d structure functions, respectively, evaluated using the leading twist “BB” parameterization of the nucleon structure functions at $Q^2 = 2$ GeV². The full calculation for g_1^d and the Bjorken limit approximation are similar, and both smaller in magnitude than g_1^N for $x \lesssim 0.8$. Here to a good approximation the deuteron structure functions are related to the nucleon structure functions by an x -independent multiplicative factor, $g_{1,2}^d \approx (1 - \frac{3}{2}P_D)g_{1,2}^N$. At larger x , one sees the onset of a classic Fermi motion effect, whereby the ratios $g_{1,2}^d/g_{1,2}^N$

increase in magnitude as $x \rightarrow 1$. In the region $0.8 \lesssim x \lesssim 1$, the finite- Q^2 results are again smaller in magnitude than those for the Bjorken limit kinematics, implying a stronger EMC effect at large x , for both the g_1^d and g_2^d structure functions.

Although the shapes of the nucleon and deuteron structure functions are very different, especially in the resonance region, when integrated over x the differences turn out to be remarkably small, particularly for the lowest moment. At large Q^2 the lowest moments of the nucleon and deuteron $g_{1,2}$ structure functions, $\Gamma_{1,2}(Q^2) = \int dx g_{1,2}(x, Q^2)$, are to a very good approximation related by:

$$\Gamma_{1,2}^d(Q^2) \approx \left(1 - \frac{3}{2}P_D\right) \Gamma_{1,2}^N(Q^2), \quad (19)$$

where for the Paris deuteron wave function the depolarization factor relating the moments is $(1 - \frac{3}{2}P_D) \approx 0.91$. Numerically, for the leading twist BB parameterization of the nucleon structure functions [38], using the Bjorken limit nucleon momentum distributions we find $\Gamma_1^{d(Bj)}/\Gamma_1^N = 0.91$ and $\Gamma_2^{d(Bj)}/\Gamma_2^N = 0.91$ at $Q^2 = 2 \text{ GeV}^2$. Using the full expressions in Eqs. (11) and (12), on the other hand, we find the ratios $\Gamma_1^d/\Gamma_1^N = 0.90$ and $\Gamma_2^d/\Gamma_2^N = 0.90$. Thus the finite- Q^2 kinematics slightly reduces the magnitude of the deuteron moments, which is consistent with the observation of the additional suppression at finite Q^2 at large x in Figs. 6 and 7.

The ratios of the deuteron to nucleon moments in the resonance region remain similar to the leading twist ratios, even though the shapes of the functions here are strongly Q^2 dependent and hence infused with large higher twist contributions. Specifically, for the MAID parameterization of $g_{1,2}^N$ the ratios at finite- Q^2 kinematics turn out to be $\Gamma_1^d/\Gamma_1^N = 0.91$ and $\Gamma_2^d/\Gamma_2^N = 0.92$, at the same $Q^2 = 2 \text{ GeV}^2$. Note that the structure functions here have been integrated from $x_{\min} \approx 0.4$ up to $x_{\max} = x_{\text{thr}}$, where $x_{\text{thr}} = Q^2/(W_{\text{thr}}^2 - M^2 + Q^2)$ corresponds to the kinematical pion production threshold, $W_{\text{thr}} = M + m_\pi$. The results using Bjorken limit kinematics differ from these only in the third decimal point.

Overall, our results suggest that for the lowest moments of the g_1 and g_2 structure functions, the nuclear effects in the deuteron can to very good accuracy be accounted for by applying the depolarization correction, $(1 - \frac{3}{2}P_D)$. This will not be true, however, for higher moments, and certainly this approximation will break down dramatically at large x , for

$$x \geq 0.7 - 0.8.$$

5. Conclusions

For most of the history of lepton–nucleus deep inelastic scattering, the discussion of nuclear effects on structure functions has been confined to analysis of high-energy data within theoretical frameworks constructed to be valid in the limit of large Q^2 and W^2 ($\gg M^2$). Recent high-quality data at lower Q^2 and W^2 , especially in the transition region where nucleon resonances merge into the DIS continuum, have revealed a richness of phenomena which had not previously been appreciated because of the lack of precision in earlier data. An accurate description and understanding of the new data clearly demands comparable advances in the theoretical tools.

Our aim in this work has been to provide the framework for analyzing modern high-precision data over the full range of kinematics where they are available. To this end we have derived relations between spin-dependent nuclear and nucleon g_1 and g_2 structure functions in the weak binding limit, which are valid at all Q^2 , and in both the traditional DIS region and the poorly explored nucleon resonance region. As a consistency check, we verify that our results approach the previously derived convolution formulas for the nuclear structure functions at large Q^2 [22].

We apply the formalism to the specific case of the deuteron, which is often used as a source of neutron structure information, and compare the size of the nuclear corrections calculated using exact kinematics and using approximations applicable at large Q^2 . We find that significant smearing of the nucleon structure functions occurs in regions where nucleon resonances are prominent, with the exact results some 10–15% smaller in magnitude than the Bjorken limit structure functions around the second and third resonance regions. The smearing is enhanced at larger x , where the full results are up to 25–30% smaller at the Δ resonance peak than those with the Bjorken limit smearing.

The enhanced smearing at finite- Q^2 kinematics is also evident for leading twist structure functions, although the effects here are less dramatic. At intermediate x the deuteron structure functions are approximately given by $g_{1,2}^d \approx (1 - \frac{3}{2}P_D)g_{1,2}^N$. However, at larger x , $0.8 \lesssim x \lesssim 1$, the finite- Q^2 results are again smaller in magnitude than those for the Bjorken limit kinematics, implying a stronger EMC effect, for both the g_1^d and g_2^d structure functions.

The ratios of the integrals of the deuteron and nucleon structure functions are to a good approximation simply related by the depolarization factor, $\Gamma_{1,2}^d(Q^2)/\Gamma_{1,2}^N(Q^2) \approx (1 - \frac{3}{2}P_D)$, even at low Q^2 ($Q^2 \sim 2 \text{ GeV}^2$). This holds for both the leading twist structure functions, and for structure functions in the resonance region, where the shapes of the functions are strongly Q^2 dependent, with deviations only of $\mathcal{O}(1\%)$.

In closing we should mention that our calculation is by no means complete. We have not considered, for example, possible modification of nucleon properties (such as masses or widths) in the nuclear medium, which in our framework amounts to neglecting nucleon dynamical off-mass-shell effects in the nucleon structure functions. Extensions in this direction can be carried out using existing models for the off-shell extrapolation [8, 21, 22, 39]; we have chosen not to do so in this work in order to more clearly isolate the effects associated with the finite- Q^2 kinematics. Similarly, we have not considered here effects beyond the nuclear impulse approximation, such as rescattering, meson exchange currents, or final state interactions. Finally, our formalism can be easily applied to other nuclei, such as polarized ^3He . This will be the subject of a future publication [29].

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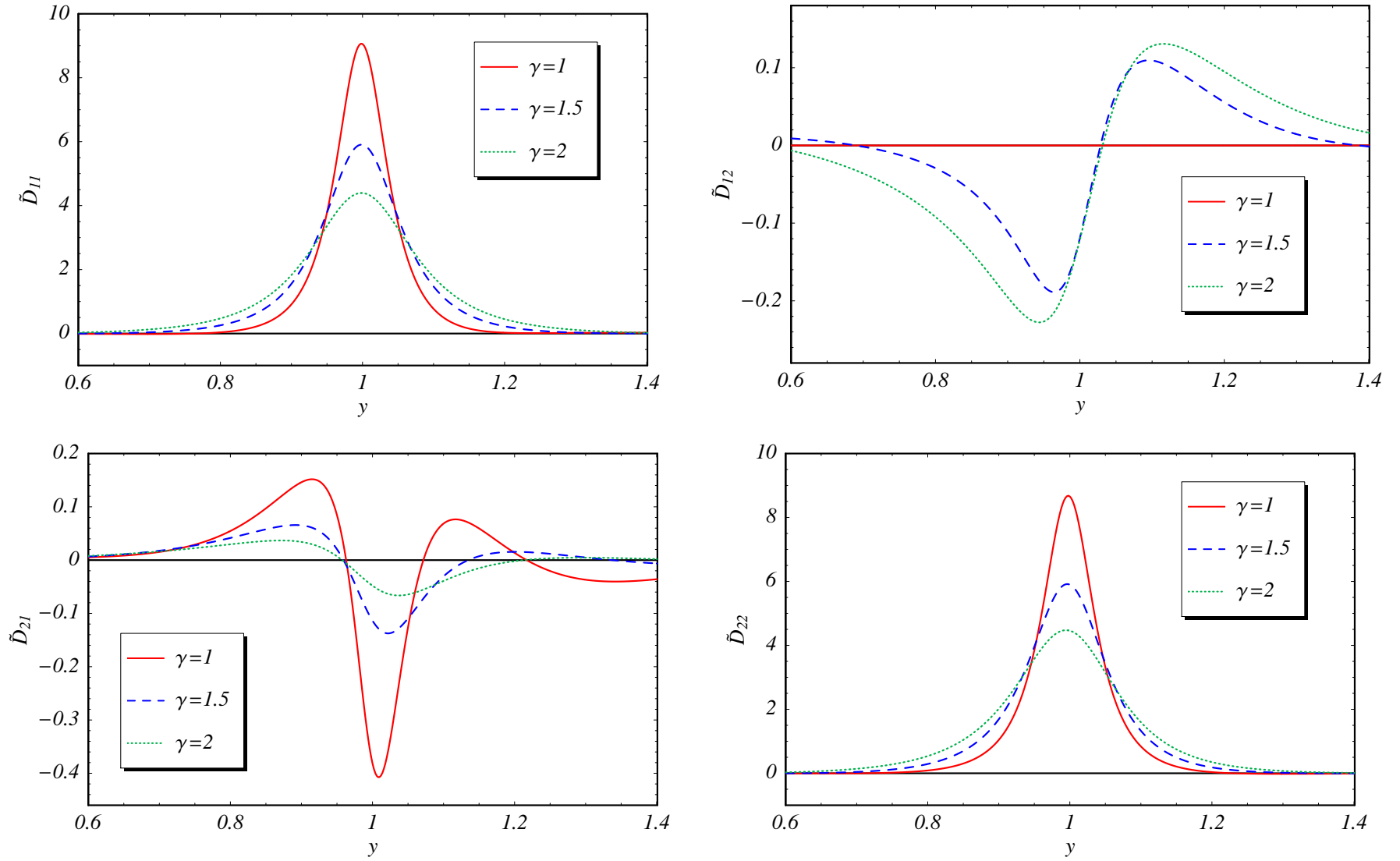


FIG. 1: Effective nucleon light-cone momentum distribution functions $\tilde{D}_{ab}(y, \gamma)$ for $\gamma = 1$ (Bjorken limit), 1.5 and 2.

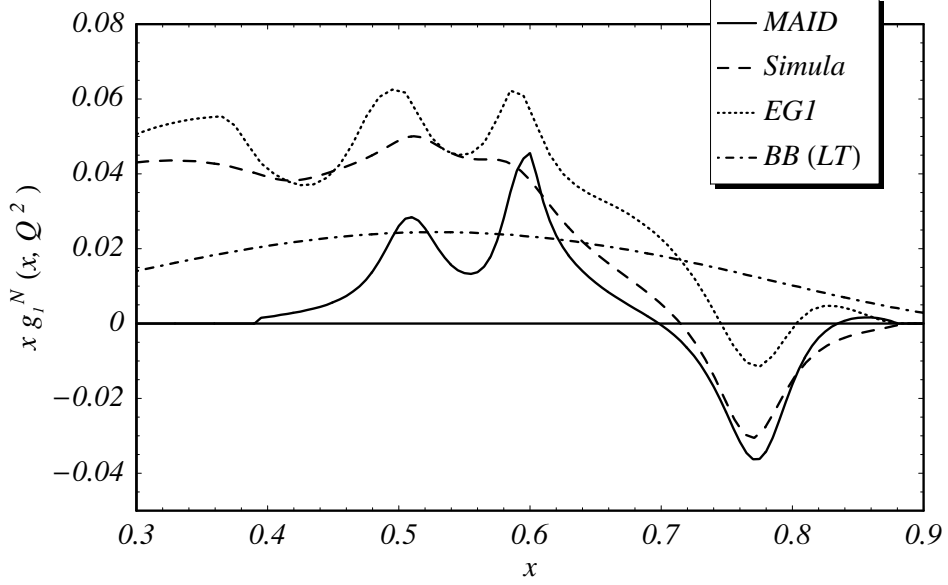


FIG. 2: Spin-dependent xg_1^N structure function of an isoscalar nucleon, from the fits of Refs. [35] (solid), [36] (dashed), [37] (dotted), and the leading twist fit of Ref. [38] (dot-dashed), at $Q^2 = 2 \text{ GeV}^2$.

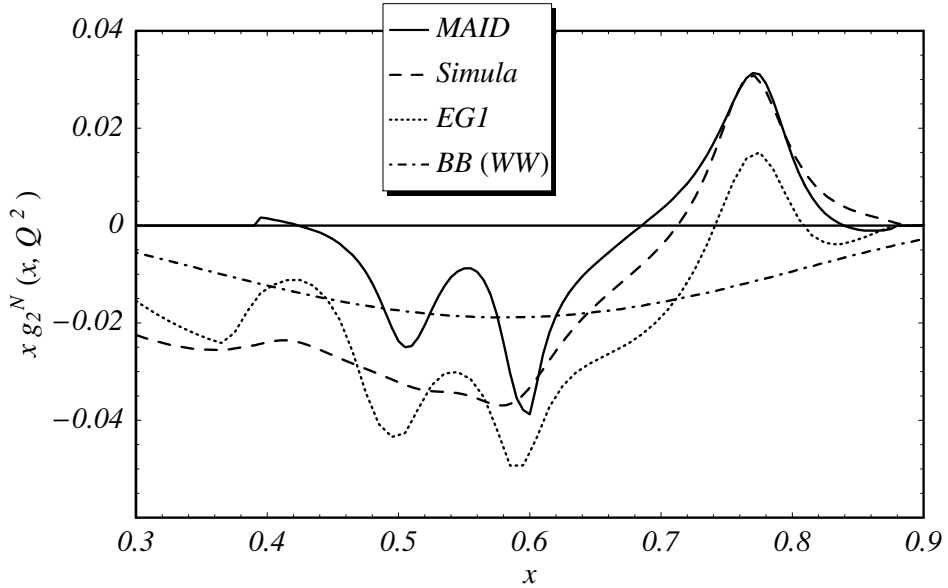


FIG. 3: Spin-dependent xg_2 structure function of an isoscalar nucleon, from the fits of Refs. [35] (solid), [36] (dashed), [37] (dotted), and the Wandzura-Wilczek approximation to g_2 using the leading twist g_1 fit of Ref. [38] (dot-dashed), at $Q^2 = 2 \text{ GeV}^2$.

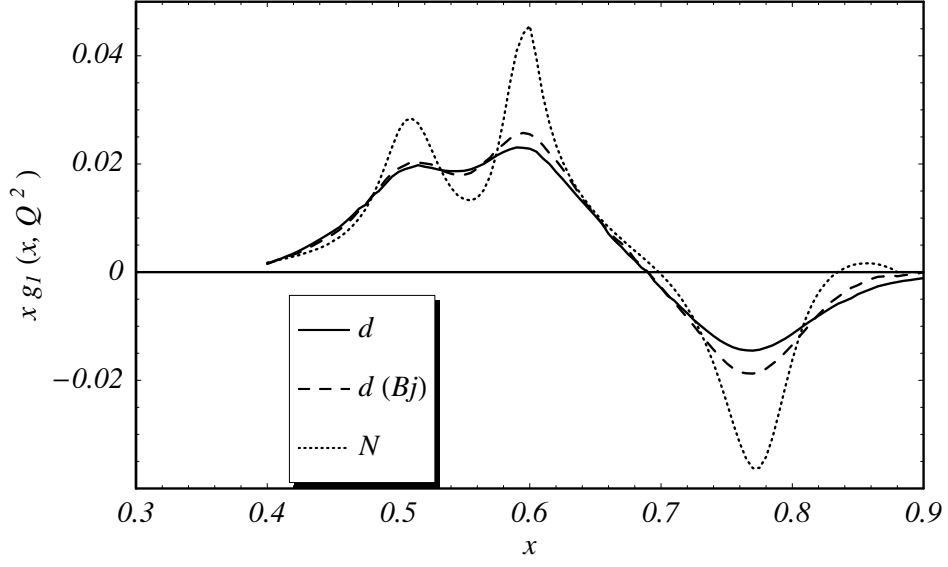


FIG. 4: Spin-dependent xg_1 structure function of the deuteron evaluated at finite- Q^2 (solid) and Bjorken limit (dashed) kinematics, compared with the nucleon (dotted) input, from Ref. [35] at $Q^2 = 2 \text{ GeV}^2$.

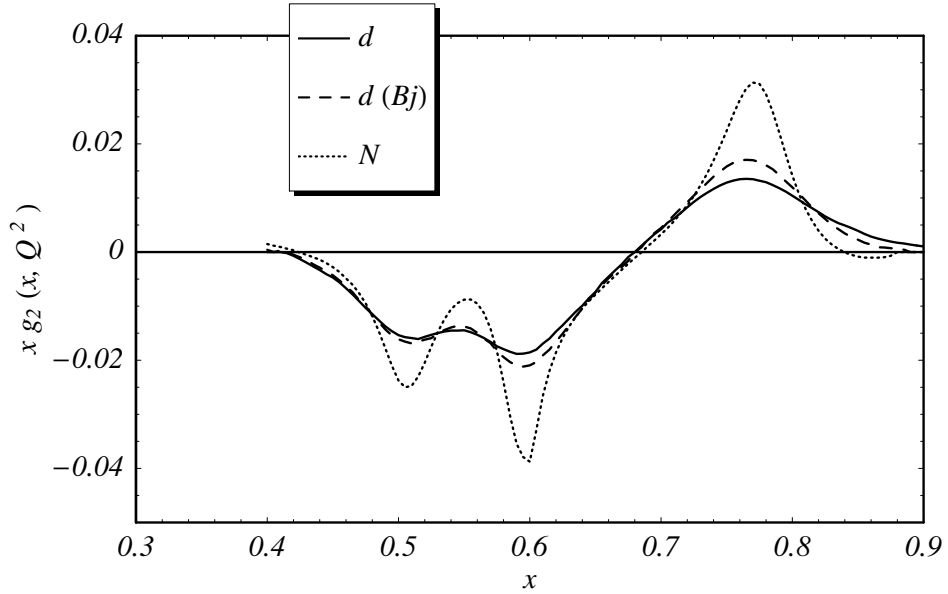


FIG. 5: Spin-dependent xg_2 structure function of the deuteron evaluated at finite- Q^2 (solid) and Bjorken limit (dashed) kinematics, compared with the nucleon (dotted) input, from Ref. [35] at $Q^2 = 2 \text{ GeV}^2$.

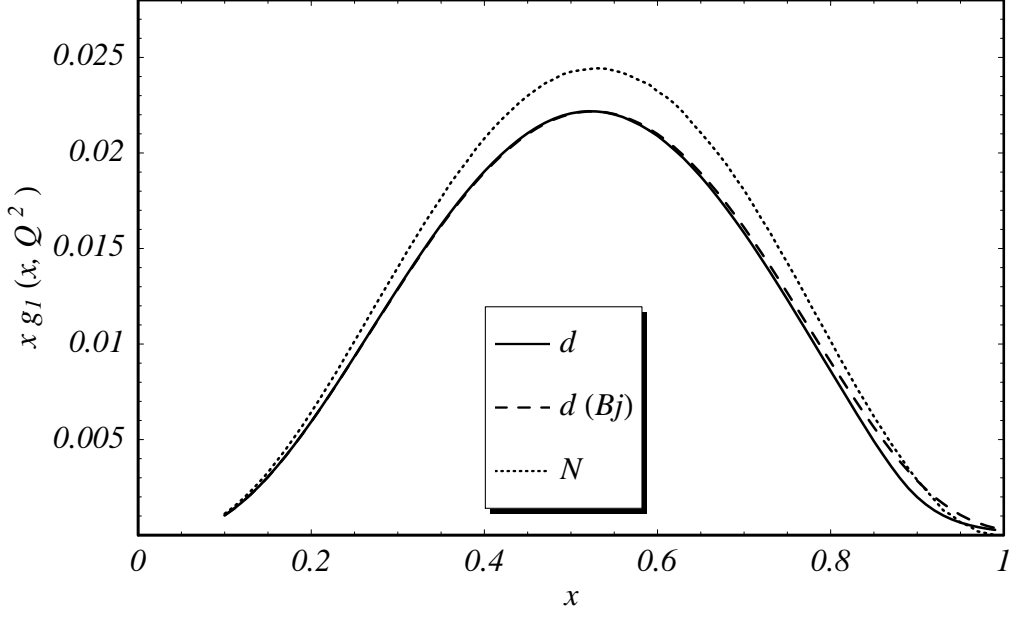


FIG. 6: Leading twist xg_1 structure function of the deuteron evaluated at finite- Q^2 (solid) and Bjorken limit (dashed) kinematics, compared with the nucleon (dotted) input from Ref. [38] at $Q^2 = 2 \text{ GeV}^2$.

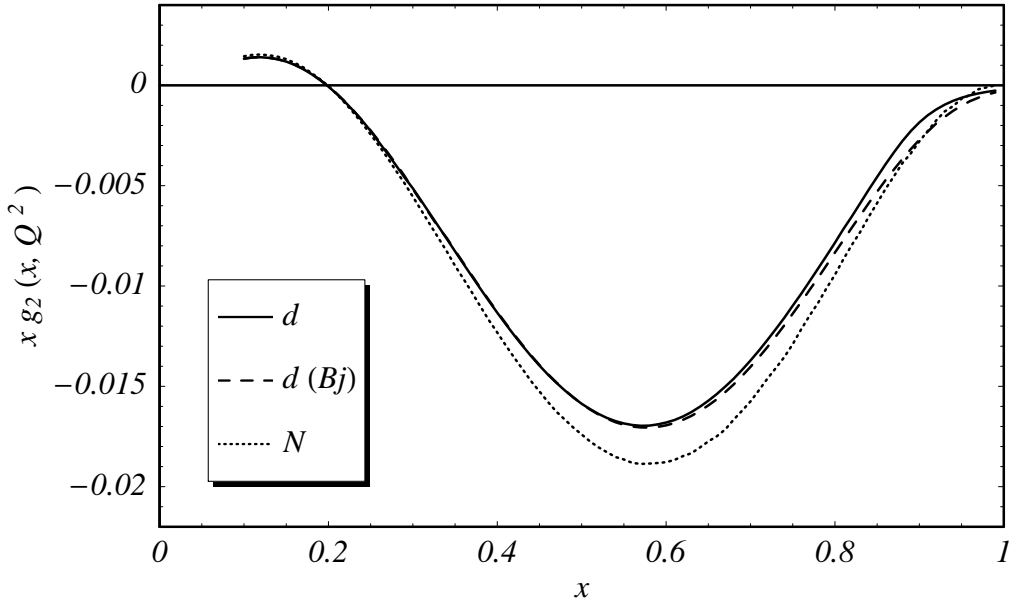


FIG. 7: Leading twist xg_2 structure function of the deuteron evaluated at finite- Q^2 (solid) and Bjorken limit (dashed) kinematics, compared with the nucleon (dotted) input from Ref. [38] at $Q^2 = 2 \text{ GeV}^2$.